Comparison of equivalent photon energy calibration methods in computed tomography

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A method of specifying the equivalent photon energy as the energy that gives the maximum correlation between linear attenuation coefficient and CT value of six standard materials, including water, was compared with standard method that specified equivalent photon energy as the energy at which water's linear attenuation coefficient is equal to the detected energy fluence averaged coefficient of water. Comparisons were made for various tube potentials, thicknesses of aluminum filtration, and water phantom thicknesses. Using the experimental data, the first method predicted changes in equivalent photon energy equal to 0.3 keV kVp^{-1}, 2.8 keV g^{-1} cm^2, and 0.75 keV g^{-1} cm^2, respectively, for the specified conditions; the precision was ±2.2 keV. Both methods estimated the same equivalent photon energies within 3 keV. This similarity was shown to be a result of the characteristics of water's attenuation coefficient. The effect of uncertainty in measured CT values and material density on the equivalent photon energy was estimated. The equivalent photon energy was used to predict CT values for high atomic number water solutions, 5 mg/ml. The difference between the measured CT values and the predicted was less than 10 CT number for elements of less than 60.

Key words: computed tomography, equivalent photon energy, CT phantoms

I. INTRODUCTION

CT values have been related to linear attenuation coefficients of materials scanned. Because the linear attenuation coefficient is only defined for a monoenergetic narrow x-ray beam, many investigators have defined an equivalent or effective photon energy which represents the x-ray beam for the condition of use. McCullough measured the transmission of water in the original EMI Mark I.[1,2] He used the water box and the scanner's detector to estimate an effective linear attenuation coefficient of water and demonstrated that the measurement was equal to the detected energy fluence averaged linear attenuation coefficient of water. He defined the equivalent photon energy as the energy at which the linear attenuation coefficient is the same as the measured effective linear attenuation coefficient. This method of determining the equivalent photon energy is the standard one against which all others can be compared. This method requires either knowledge of the detected energy spectrum or a direct measurement of the transmission of water and as a consequence it has not been universally used to specify the equivalent photon energy.

A more empirical approach is that the equivalent photon energy is the photon energy that describes a linear relationship between the CT values and linear attenuation coefficients of several materials. Rutherford *et al.*[3] used a series of hydrocarbons to obtain this equivalent photon energy. Dubal and Wiggli[4] suggested four substances whose energy gradient, $d\mu/dE$, differs as much as possible: air, water, magnesium, and an arbitrary fourth substance. Millner *et al.*[5] suggested yet another set of materials and an empirical approach for the determination of the equivalent energy. In their approach a water-filled phantom containing five plastics was scanned, and the CT values obtained for the plastics and water were correlated with their linear attenuation coefficients at various energies. The energy resulting in the maximum correlation coefficient was specified as the equivalent photon energy. This particular method has the advantage that it uses the AAPM CT phantom, which is widely distributed throughout the world.[6] Each plastic in the commercial realization of the AAPM CT phantom is from single batch[7] and therefore batch to batch variation of composition and physical density of the plastics will not introduce errors in comparison of equivalent photon energies of various scanners performed with different phantoms. This study compared the method proposed by Millner *et al.* with the standard method as described by McCullough. The comparison was as function of tube potential, source filtration, and phantom size.

Other approaches to equivalent photon energy selection[2,8-10] have been suggested but are not the subject of this paper.

A. Experimental

The AAPM CT phantom[6] containing its contrast linearity insert was scanned on both an Ohio-Nuclear Δ-2010 and Δ-2020 scanner. The insert contained five different plastic pegs (Table I) positioned in a circle of 10 cm diameter about the center. The CT number for each plastic peg and for water was obtained, the water value being measured at a corresponding distance from the center. Each operating condition

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TABLE I. Properties of AAPM CT phantom contrast linearity insert.

Material	Elemental composition	Density (g cm^{-3})
Water	H_2O	1.00
Plexiglas	$C_5H_8O_2$	1.19
Lexan	$C_{12}H_{12}O_3$	1.20
Nylon	$C_6H_{11}N_{10}$	1.15
Polystyrene	C_8H_8	1.05
Polyethylene	C_2H_4	0.94

was calibrated independently, setting water to zero and air to −1000. This procedure is performed by the manufacturer as part of their calibration of the scanner.

Scans were obtained with the Δ-2010 scanner of the insert in the water-filled phantom for several tube potentials in the range of 80–138 kVp with 4 mm aluminum filtration with additional scans at 120 kVp for 6 and 9 mm aluminum filtration. To evaluate the effect of patient thickness, scans were obtained at 120 kVp and 4 mm aluminum filtration for six phantom thicknesses with the Δ-2010 and four phantom thicknesses with the Δ-2020 (Table II). The plexiglass of the resolution ring and teflon of the bone ring were treated as water with the density of those substances. The water thickness of the water-filled ham container was estimated by the average of the maximum and minimum cross-sectional thickness.

B. Theoretical

Mass attenuation coefficients for the plastics and water were calculated using their elemental mass fractions and tabulated mass attenuation coefficients[11] for the elements at several energies. Linear attenuation coefficients were then obtained at any energy from log-log interpolation of the calculated mass attenuation coefficients using the nominal densities listed in Table I.

The detected energy spectra averaged attenuation coefficient of water was calculated as

$$\bar{\mu}_w = \int f(E)\mu_w(E)dE, \tag{1}$$

and similarly the detected energy spectra weighted difference in attenuation coefficient ($\overline{\Delta\mu}$) between a material and water was calculated as

$$\overline{\Delta\mu} = \int f(E)[\mu_m(E) - \mu_w(E)]dE, \tag{2}$$

where E is the photon energy, $f(E)$ is the normalized detected energy spectra, and $\mu_m(E)$ and $\mu_w(E)$ are linear attenuation coefficients. The formula to calculate the detected spectra, $f(E)$, has previously been published.[12] For this study detector sensitivity variation was neglected.

The changes of CT values for materials of interest were calculated using the equivalent photon energy by the following:

$$\Delta H = 1000\,\frac{\Delta\mu(E\text{ equiv})}{\mu_w(E\text{ equiv})}, \tag{3}$$

or calculated, assuming that they are linearly related to the detected energy fluence average difference in attenuation coefficients (Eq. 2) by:

$$\Delta H = 1000\,\frac{\int f(E)[\mu_m(E) - \mu_w(E)]dE}{\int f(E)\mu_w(E)dE}. \tag{4}$$

Equation (4), similar to relationships expressed by several investigators, has been shown[12] to predict CT values under general assumptions of a linear reconstruction and an arbitrary hardening correction.

C. Equivalent energy methods

The method described by Millner *et al.*[5] will be called the maximum correlation method. The measured CT values obtained for water and the plastics in the contrast linearity insert (Part A) were linearly regressed with their linear attenuation coefficients over a range of photon energies to determine the energy resulting in the maximum correlation. This energy was designated the equivalent photon energy.

The method described by McCullough[1,2] will be called the water attenuation method. This method determines the equivalent photon energy by matching water's monoenergetic linear attenuation coefficient with its calculated (Eq. 1) detected energy fluence averaged linear attenuation coefficient. The energy at which this match occurred was designated as the equivalent photon energy.

The effect of random variation of densities and CT values of the plastics and water on the equivalent photon energy predicted by the maximum correlation method was evaluated. CT values for the plastics and water were predicted for the 21.8 cm thickness phantom scanned at 120 kVp and 4 mm Al using Eq. (4). The effect of density variation on equivalent photon energy was assessed by modifying one at a time by 1% the assigned densities used to determine the linear attenuation coefficients and then correlating the new coefficients with the CT values. The effect of CT number variation on equivalent photon energy was similarly evaluated. The predicted CT value for an individual material was modified by 10 CT units and substituted into the correlation to determine a new equivalent photon energy. The overall effect of uncertainty

TABLE II. Phantom thickness scanned.

				Insert + water-filled phantom plus			
	Insert only	Insert plus resolution ring	Insert alone	Resolution ring	Bone ring	Resolution plus bone ring	Insert plus water-filled ham container*
Δ-2010		X	X	X	X	X	X
Δ-2020	X	X	X	X			

* Tupperware Corp. Orlando, FL.

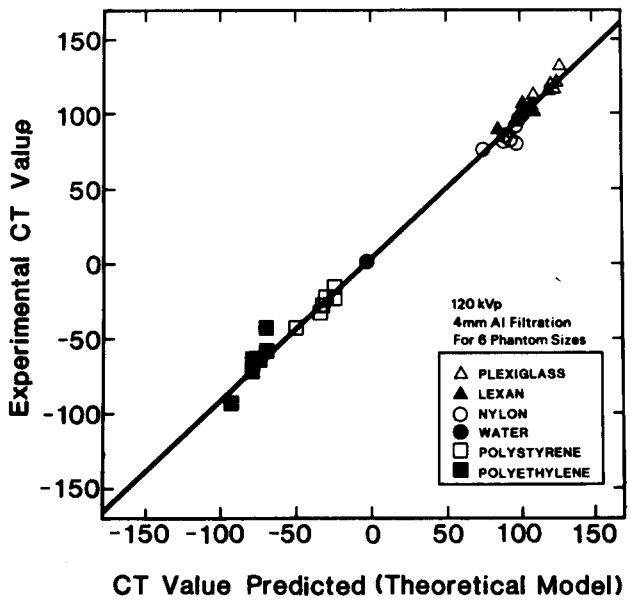

FIG. 1. Experimental CT values measured for five plastics and water vs predicted CT values [Eq. (4)] for six phantom thicknesses. Scans were made at 120 kVp and 4 mm Al filtration. Measured CT values were adjusted for $H_{water} = 0$.

in density and CT number was calculated as the root sum square of the individual uncertainties.

II. RESULTS

The predicted CT values with Eq. (4) and the experimental values for various phantom thicknesses were compared. Figure 1 demonstrated the ability to predict CT values as a function of phantom thickness for the five plastics in the insert. The water value was subtracted from the measured CT values, because the calibration procedure is valid for only one phantom size. Yet Eq. (4) remains valid if the measured CT value of water is subtracted from the measured CT value of the plastic. The solid line represents the linear least squares fit to the data with a resulting correlation of 0.9978.

Figure 2 shows the results on the Δ-2010 that are characteristic of the maximum correlation method. The correlation coefficient rapidly increases with increasing photon energy up to a maximum and then declines more slowly beyond the maximum. The photon energy corresponding to the maximum correlation, i.e., the equivalent photon energy, increased from 59.8–86.4 keV as the effective water thickness of the phantom increased from 10.7–34.0 cm. The value of the maximum correlation coefficient, however, declined with increasing phantom thickness. The combined curve represents the correlation and equivalent photon energy for the range of thicknesses. The CT values for each thickness were adjusted such that water equals zero. The overall correlation is still quite high.

Figures 3–5 present the change in equivalent photon energy found as a function of tube potential, filtration thickness, and phantom thickness, as well as experimental data points. The solid line is the equivalent photon energies predicted by the maximum correlation method utilizing CT values predicted by Eq. (4), while the dashed line is the equivalent photon energies predicted by the water attenuation method.

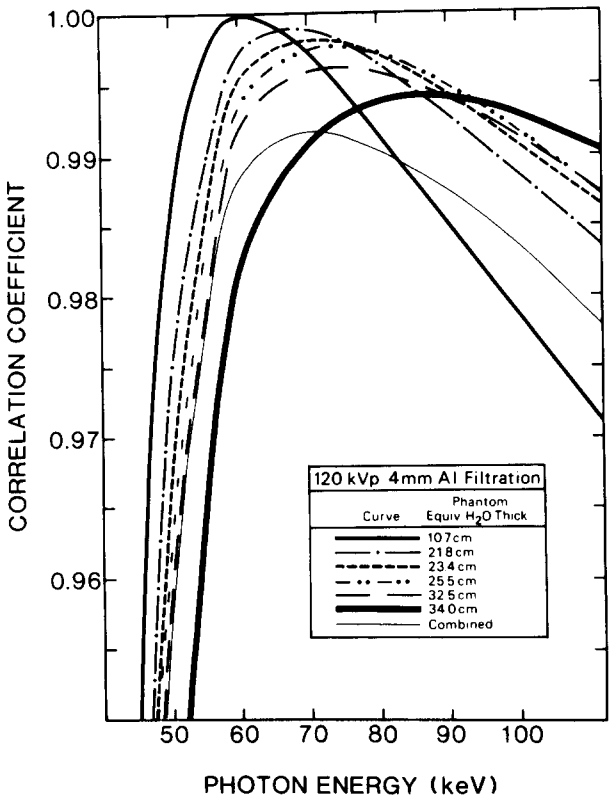

FIG. 2. Correlation coefficient as a function of photon energy for six equivalent water thicknesses scanned at 120 kVp and 4 mm Al. Correlation coefficient is determined from linear correlations of measured CT values of five plastics and water with their linear attenuation coefficients. Combined curve represents results for the range of thicknesses ($H_{water} = 0.0$).

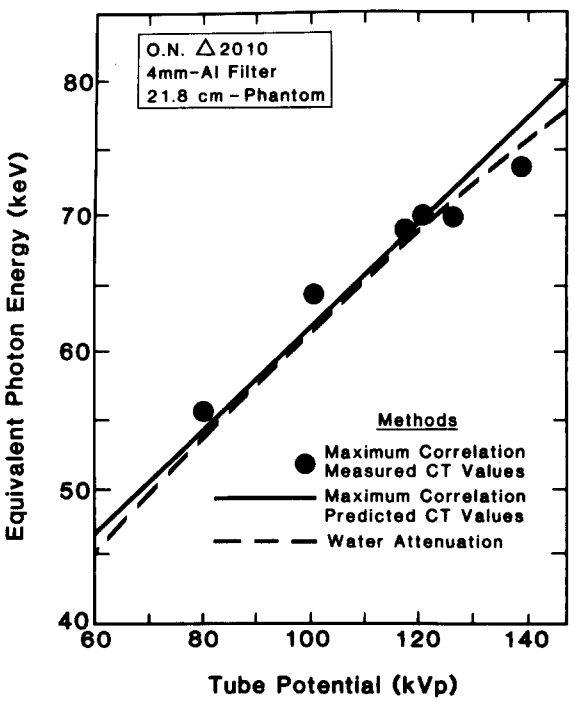

FIG. 3. Equivalent photon energy as a function of tube potential for 4 mm Al filtration. Data points were determined using the maximum correlation method with measured CT values. The solid line was determined using the maximum correlation method with CT values predicted by Eq. (4). The dashed line was determined using the water attenuation method.

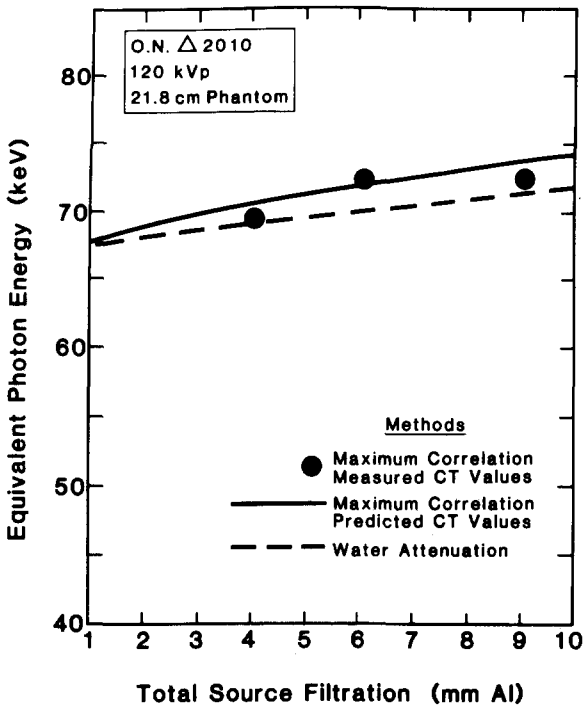

FIG. 4. Equivalent photon energy as a function of filtration for 120 kVp tube potential. Data points were determined using the maximum correlation method with measured CT values. The solid line was determined using the maximum correlation method with CT values predicted by Eq. (4). The dashed line was determined using the water attenuation method.

Both lines are based on the same computer model for the x-ray spectrum and attenuation by the phantom. The error bar (±2.2 keV) for the ON Δ-2020 data point (Fig. 5) is two standard deviations based on 18 independent measurements.

Figure 3 shows the similarity between the two equivalent photon energy methods at various tube potentials. The maximum difference in predicted equivalent photon energy between the two methods was 3.0 keV. Over the experimental range of 80–138 kVp increases in tube potential resulted in equivalent photon energy changes of 0.3 keV kVp^{-1}.

For a fixed 120 kVp tube potential, source filtration thickness changes (Fig. 4) resulted in equivalent photon energy changes of 2.8 keV g^{-1} cm^2. Over the range of 4–9 mm Al this amounted to a relatively small change in equivalent photon energy. The degree of change was of the order of the difference in equivalent photon energy predicted by the two methods and of the order of the two standard deviation value discussed above.

Phantom thickness (Fig. 5) significantly affected (0.75 keV g^{-1} cm^2) the predicted equivalent photon energy over the range of experimental thicknesses (2.5–34.5 cm) for 120 kVp tube potential and 4 mm Al filtration. From Figs. 4 and 5 it can be seen that the water attenuation method showed less dependence (smaller slope) on filtration either by aluminum or phantom thickness than the maximum correlation method.

The effects of uncertainty of density and CT values on the equivalent photon energies determined by the maximum correlation method are shown in Table III. Water had the most significant effect accounting for 84% of the variance of the equivalent photon energy; Plexiglas had a negligible effect.

III. DISCUSSION

Both the maximum correlation method and the water attenuation method predict similar equivalent photon energies. This can be explained by inspection of the linear attenuation coefficient as a function of photon energy for water and for the five plastics utilized (Fig. 6). The plastic attenuation coefficient values are nearly parallel while the shape of the water curve differs significantly. The more rapid decrease of water's attenuation coefficient relative to the plastic values reflects its different photoelectric contribution at lower energies.

The effect of these relative changes on the relationship between the CT value and linear attenuation coefficient is shown in Fig. 7. These curves represent measured CT values for plastics and water as a function of linear attenuation coefficient at five selected photon energies. Two regression lines are shown for each photon energy—one includes the five plastics and water in the regression (solid), the other includes only the five plastics (dashed). At 70 keV the two lines come very close to each other. At photon energies different from 70 keV, however, the effect of the water value on the regression line is quite evident, resulting in smaller linear correlation coefficients. On the other hand, the curves without water demonstrate a high correlation coefficient over the entire energy range (40–80 keV). Thus as the photon energy changes from the highest correlation coefficient, the water

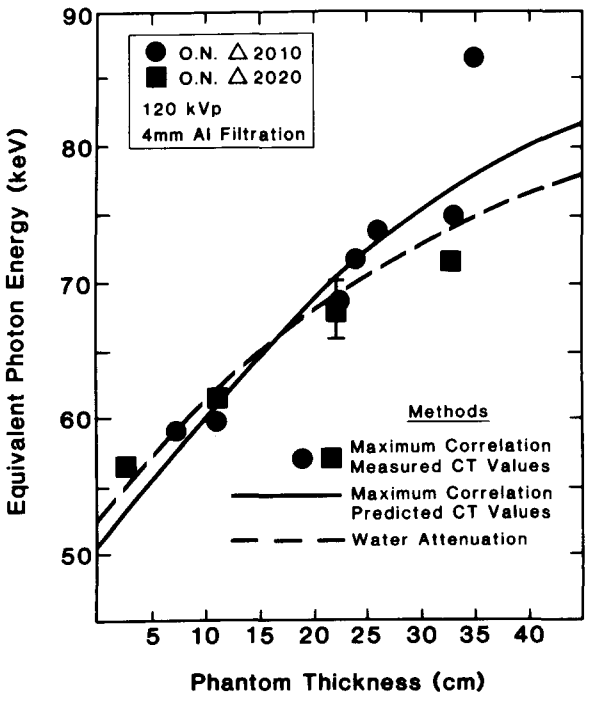

FIG. 5. Equivalent photon energy as a function of phantom thickness for 120 kVp and 4 mm Al filtration. Data points were determined using the maximum correlation method with measured CT values. The solid line was determined using the maximum correlation method with CT values predicted by Eq. (4). The dashed line was determined using the water attenuation method.

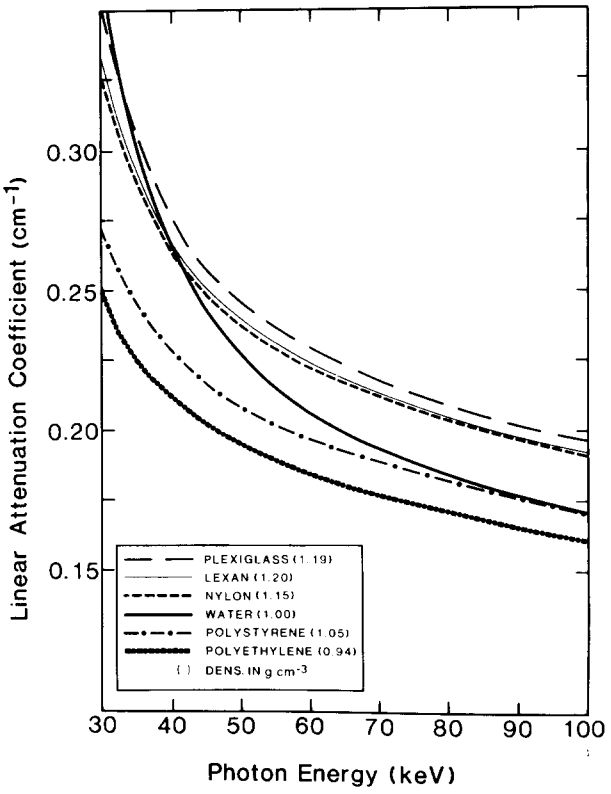

FIG. 6. Linear attenuation coefficients of five plastics and water as a function of photon energy. Densities in parenthesis were assumed.

CT value diverges from a linear relationship, producing the characteristic effect utilized in the maximum correlation method.

This effect can be seen quite clearly in Fig. 8, which shows the curve with water (thick lines) at three tube potentials—80, 100, and 120 kVp. Although each curve has a similar shape, each tube potential results in a peak correlation at a different photon energy. Without water in the correlation, however, the resulting curves (thin lines) show very little difference in the correlation, a large broad peak, and a peak value that is virtually identical for all three tube potentials.

Table III, which shows the effects of variation in density and CT value on equivalent photon energy, also demonstrates the significance of water. A variation in the water value produces the largest variation in the equivalent photon energy.

Hence the similarity of the resulting equivalent photon energy predicted by the maximum correlation method and the water attenuation method is not surprising. The maximum correlation method relies strongly on water's influence and its changing characteristics relative to the five plastics, of similar chemical composition, for choosing the equivalent photon energy.

The decreasing correlation for larger phantom thicknesses demonstrated in Fig. 1 has not fully been explained. A possible explanation is that the decrease might be due to an increase in the variance of CT values for the larger thicknesses. But this explanation is not supported by the measurements for filtration where the correlation remained high. Some preliminary measurements show that a change in the slice

TABLE III. Sensitivity of maximum correlation method.

Material	Change in effective energy (KeV) due to: 1% Density change	10 CT number change
Plexiglas	± 0.1	± 0.1
Lexan	± 1.3	± 1.0
Nylon	± 2.3	± 1.9
Water	± 9.7	± 9.6
Polystyrene	± 2.3	± 2.2
Polyethylene	± 2.3	± 2.5
Root sum square total (less water)	± 10.57 ± 4.2	± 10.39 ± 4.0

thickness results in a change in the equivalent photon energy, suggesting that this decreasing correlation may be associated with an increase in detected scattered radiation as the phantom thickness increases.

The ability of the equivalent photon energy to predict CT values depends on the particular application. Consider, for example, the prediction of CT values of higher atomic number materials having a K-edge show value is significant when compared with the spectral distribution. The effect of this edge on the CT value is not considered by equivalent photon energy methods because such methods assume only a single energy photon is present.

Figure 9 shows the measured CT values for ten dilute solutions containing high atomic number elements. The details of these measurements have been reported previously.[10] Two predictions of CT values are also shown. The solid line represents an equivalent photon energy prediction of the values. The predicted CT values continue to rise with increasing atomic number until a material with a K-edge just above the equivalent photon energy is reached, at which point the predicted CT value drops to a low value.

The measured CT values and the CT values predicted using Eq. (4) (dashed line) do not show a sharp change, but rather a gradual peaking and subsequent decline in CT values

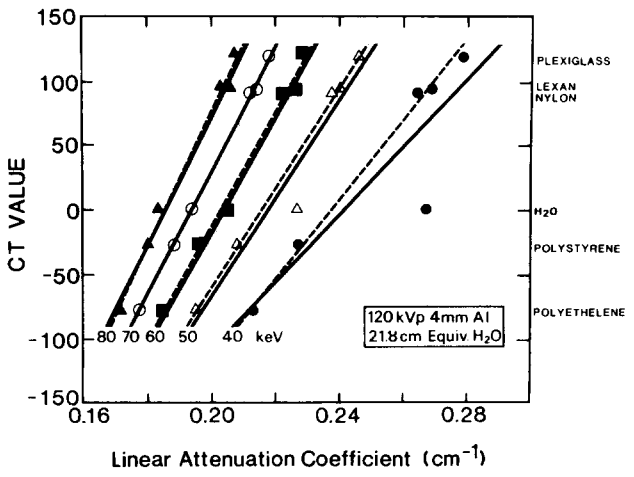

FIG. 7. CT value predicted by Eq. (4) vs linear attenuation coefficient for five different photon energies. Solid line describes linear regression for five plastics and water. Dashed line describes regression for five plastics only.

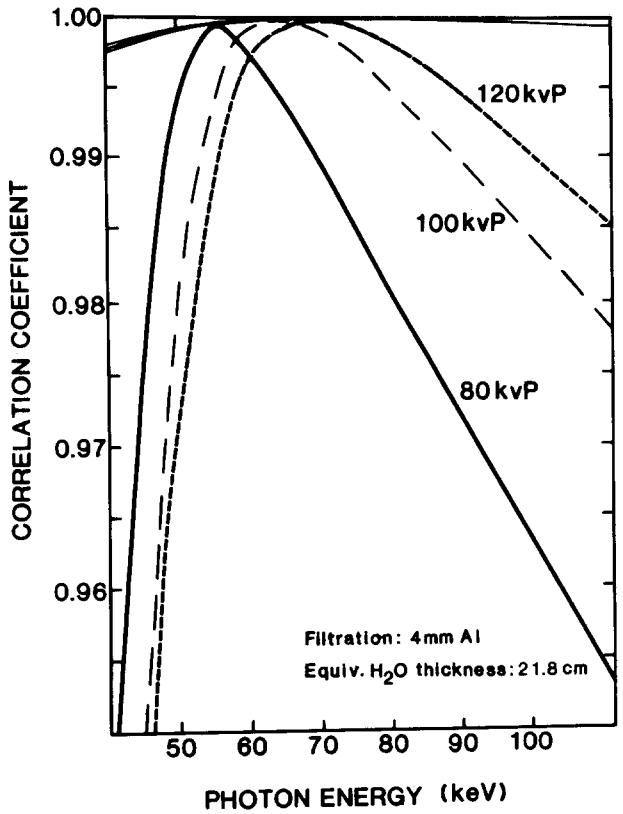

FIG. 8. Correlation coefficient from linear regression of measured CT values for five plastics and water with their linear attenuation coefficient as a function of photon energy. Three tube potentials 80, 100, and 120 kVp are shown. Thick lines represent this correlation. Thin lines represent correlation without water.

at an atomic number whose K-edge is significantly lower than the specified equivalent photon energy. The smooth shape reflects the gradual spilling over the K-edge of the x-ray beam photons until the energy fluence averaged attenuation coefficient causes a decrease in the CT value.

The shape of the equivalent photon energy prediction curve follows the trend of measured CT values up to the maximum measured value. Up to atomic number 60 the difference between the CT values predicted using the equivalent photon energy, Eq. (3), and the CT values predicted with a complete x-ray spectrum, Eq. (4), is less than 10 CT numbers. This difference is less than 1% for the absolute of the linear attenuation coefficient.

The ability to use the equivalent photon energy, Eq. (3), to predict CT values of bone was studied. The CT values of water volumes containing various amounts of bone (0–1.0 mass fraction) were predicted both by Eqs. (3) and (4). The equivalent photon energy was estimated using the water attenuation method for a 120 kVp x-ray beam filtered by 4 mm Al. The phantom size was 20 cm. The difference of CT values corresponded to less than 2% error in the mass fraction bone. This supports suggestions of Kijewski and Bjärngard[11] that a single-energy CT scan can be used to obtain information needed for radiotherapy dose calculations. This error is also comparable to the precision of bone density measurements performed with CT.[12]

IV. CONCLUSIONS

While CT values can be related to their linear attenuation coefficients at some equivalent photon energy, the sensitivity of equivalent photon energy to the many physical parameters suggests caution when specifying this energy. The most important of these factors and the one we have least control over is object thickness. Our results are equivalent to the change expected between a cross section through the lungs and a cross section through the abdomen. This suggests that the accuracy of predicted CT values *in vivo* in a nonwater-bath scanner for any thickness is not that suggested by a CT scan of plastics on a single thickness phantom. Although equivalent photon energies can be specified with a high correlation, for a range of object thicknesses, its use for a single thickness, especially near the end of the range of object thicknesses, may be inappropriate.

The maximum correlation method predicts an equivalent photon energy similar to the water attenuation method. The characteristics of water's attenuation coefficient on the maximum correlation method produce this similarity. Thus the maximum correlation method, a simple empirical method, can predict an equivalent photon energy as useful as that obtained using the water attenuation method, without the detailed knowledge of the detected energy spectra or a special measurement method by the latter method.

If detailed knowledge of the detected energy spectra is available, the application of Eq. (4) is more appropriate and no more difficult. This theoretical Eq. (4) approach accu-

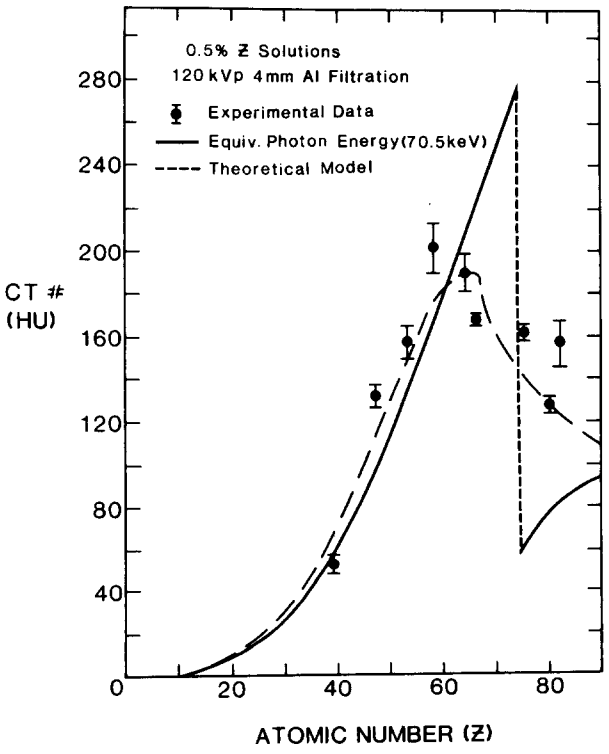

FIG. 9. CT number (Hounsfield Unit) as a function of atomic number for 0.5% solutions of high atomic number materials. Points represent experimental data (Ref. 10). Solid line represents predicted CT values by an equivalent photon energy of 70.5 keV and the dashed line represents predicted CT values using Eq. (4).

rately predicted CT values for the five plastics and predicted the CT values for weak solutions of high atomic number materials better than either equivalent photon energy method.

Equivalent photon energy methods are useful over some range of parameter variations. Their usefulness must be determined by the task. Over a range of higher atomic number solutions, the equivalent photon energy method resulted in large errors in predicted CT values, even for atomic number materials whose K-edge is significantly below the specified equivalent photon energy. For lower atomic number materials, however, the equivalent photon energy method can provide a useful, fairly accurate approach to predicting CT values.

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