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Effects of the Polyenergetic Character of the Spectrum of ^{125}I on the Measurement of Bone Mineral Content

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The effects of the polychromatic nature of the spectrum of ^{125}I on the photon absorptiometric determination of bone mineral content (BMC) have been investigated. Theoretical analysis of the effects was performed assuming exponential attenuation of the individual components of the spectrum. These analyses were in good agreement with, and verified by, experimental results on phantoms. The photon absorptiometric determination of BMC was found to be nonlinearly related to the actual thickness, g/cm², of hydroxyapatite. However, a linear relation between these two variables was derived which yielded values of BMC within ±1% of those predicted by the nonlinear relation over a range of 0.6 to 1.5 g/cm² of hydroxyapatite. Variations in soft-tissue cover were found to alter the determination of BMC by 0.5 to 1.0% per g/cm² of soft tissue, assuming a tin filtration of 51 μm of the ^{125}I source. The effects of tin filtration of the primary beam were investigated for filter thicknesses over the range of 0 to 76 μm of tin. Calculations, verified by measurements on phantoms, indicated that variations in the amount of tin filtration altered the value of BMC by 0.12 to 0.16% per μm of tin. Maintaining a tolerance of ±2.5 μm on the thickness of tin filtration of ^{125}I sources used in BMC measurements would limit the variation in BMC due to the variation of filter thickness to ⩽1%.

Key words: absorptiometry, bone, ^{125}I, filtration, hardening.

AS A POLYENERGETIC BEAM OF RADIATION passes through matter the low energy components of the beam are usually preferentially attenuated with respect to the high energy components. This effect is termed "hardening" since the increasing proportion of the high energy components results in more penetrating or "harder" radiation. The spectrum of ^{125}I, the radionuclide most extensively used for photon absorptiometric measurements of bone mineral content (BMC), consists of five photon energies ranging from 27.2 to 35.4 kev. Although this spectrum can be made more nearly monoenergetic by filtration with tin, which has a K-absorption edge at 29.2 kev, variation of the thickness of tin filtration among sources alters the measured BMC.[11]

The effect of hardening on BMC measurements is such that the measured bone mass does not increase linearly with the actual bone mass. That is, values of BMC measured for bones of large mass are less than the values which would be obtained by linear extrapolation from measurements of bones of smaller

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mass. Such an extrapolation presupposes that the photon source exhibits exponential attenuation — a condition requiring that the photon source be monoenergetic, and the measurements be made in a narrow-beam geometry.

In developing the theory of the improved method of bone mineral measurement by the photon absorptiometric method, Cameron and Sorenson[2] assumed that the photon source was essentially monoenergetic, i.e., that the attenuation of its radiation was exponential. In a later article, Sorenson and Cameron[9] reported that an ^{125}I source filtered with 0.1 mm of tin did show exponential absorption over three decades of attenuation. In the same report, there was also a discussion of the observation of a slight nonlinearity in the relationship between mineral content measured by photon absorptiometry and ash weight of the measured bone section. This nonlinearity was attributed partly to the finite cross-section of the photon beam. Effects on the BMC measurements associated with beam geometry have been discussed elsewhere,[5] and those associated with the nature of the ^{125}I spectrum are the subject of this paper.

The errors affecting the accuracy and precision of the BMC measurement which are traceable to the spectral composition of ^{125}I are on the order of 1%. Although the clinical precision of the technique is typically 2 to 3%,[2,8] investigators have achieved a precision of <1% in clinical situations by careful control of the conditions of the measurement.[6, 10] Such precision is desirable for studying the natural history of demineralization attributable to aging and disease which proceeds at the rate of ~1% per year.[8] However, it should be understood that the equations presented here to determine BMC were derived from considerations of the polyenergetic character of the spectrum of ^{125}I and do not include effects attributable to source-detector geometry, finite beam size, counting statistics, or patient repositioning. The authors do not propose that these equations should be used to determine BMC in conjunction with any particular measurement system.

Theoretical Estimate of the Hardening Effect

In making a determination of the BMC by the Cameron-Sorenson technique[2] two measurements are required: the count rate I_0^* through a thickness t_{ST} of soft tissue only and the count rate I through the thickness t_{BM} of bone mineral plus the thickness $(t_{ST} - t_{BM})$ of soft tissue. The total thickness of soft tissue plus mineral is constant for all points of measurement. With this assumption and the further assumption that the photon source exhibits exponential attenuation, the quantity $\ln (I_0^*/I)$ can be shown to be linearly related to the mass of bone mineral in the path of the photon beam.

The attenuation of a beam consisting of a discrete spectrum of radiation can be generally described by the equation

$$I = I_o \sum_{i=1}^{n} f_i \exp\left\{-\sum_{k=1}^{m} \mu_{k,i}\rho_k t_k\right\}$$

where I = the intensity of the radiation after attenuation,

I_o = the unattenuated intensity of the radiation,

n = the number of components in the radiation spectrum,

f_i = the fraction of photons in the beam having the energy of the i^{th} spectral component,

m = the number of substances in the beam path,

$\mu_{k,i}$ = the mass attenuation coefficient of the k^{th} substance at the energy of the i^{th} spectral component,

ρ_k = the density of the k^{th} substance,

t_k = the thickness of the k^{th} substance.

Considering the ^{125}I spectrum to consist of five discrete components, and defining the mass per unit area, M, of a substance as the product of its density, ρ, and its thickness, t, leads to a relation for $\ln (I_o^*/I)$ as a function of M_{BM} and M_{ST} obtained from the previous equation, which is

$$\ln(I_o^*/I) = \ln\left\{\frac{\sum_{i=1}^{5} f_i \exp\{-\mu_{ST,i} M_{ST}\}}{\sum_{i=1}^{5} f_i \exp\{-\mu_{ST,i}M_{ST} + [(\rho_{ST}/\rho_{BM})\mu_{ST,i} - \mu_{BM,i}] M_{BM}\}}\right\} \quad (1)$$

where the subscripts ST and BM refer to soft tissue and bone mineral, respectively. Hereinafter I_o^* will refer to the count rate measured for a beam transmitted through a certain thickness of soft tissue (water) only, and I will refer to the count rate measured for a beam transmitted through an identical total thickness of soft tissue plus mineral (aluminum or hydroxyapatite).

The mass attenuation coefficients used in Equation 1 were obtained from interpolations of published values.[4] Tabulations of the fractional composition of the ^{125}I spectrum have also been published.[3] The values used for the calculations in this paper are shown in Table 1.

The mass attenuation coefficients for the aluminum alloy and water were independently determined for the K_α radiation of ^{125}I. The Ross-balanced filters technique[7] was used to isolate the $K_{\alpha 1}$ and the $K_{\alpha 2}$ photon energies from the spectrum by alternately filtering the beam with copper and tin. Attenuation curves were determined which yielded K_α attenuation coefficients of $2.047 \pm .014$ (SD) cm^2/g for the aluminum and $0.4245 \pm .0003$ (SD) cm^2/g for water. Similar values calculated from the theoretical $K_{\alpha 1}$ and $K_{\alpha 2}$ attenuation coefficients given in Table 1 were 2.048 cm^2/g and 0.4209 cm^2/g for aluminum and water, respectively. The coefficients determined by the two methods agree to within 1%.

Experimental Estimate of the Hardening Effect

To demonstrate the validity of Equation 1 for predicting the effect of hardening due to the mineral component of the system on the measured value of $\ln(I_o^*/I)$, measurements were made on an aluminum step-wedge. The wedge, which was made from 2024 aluminum alloy, had eight steps ranging in thickness from 0.164 cm to 1.593 cm. The elemental composition and density of the alloy were obtained from the manufacturer's handbook.[1] Measurement of the density of the wedge agreed with the tabulated value for the alloy, which is 2.77 g/cm^3. The wedge was immersed in 4 cm of water; however, the effective water thickness was 4.75 cm including the lucite water container and support platform. The effect of hardening due to soft tissue surrounding the absorber was determined by measuring $\ln(I_o^*/I)$ for a 0.5 cm thick section of bovine bone covered with various thicknesses of lucite.

The variation of $\ln(I_o^*/I)$ with the thickness of tin filtration in the ^{125}I beam was determined by performing the transmission measurements on the stepwedge using from zero to three tin filters in the beam. Each filter was cut from the same commercially prepared sheet of tin foil of thickness 0.001 inch with a batch-to-batch tolerance

TABLE 1. Values Used in Theoretical Calculations of Hardening of ^{125}I Spectrum

Component and energy (kev)	Fraction of spectrum	Mass attenuation coefficients (cm^2/g)			
		Aluminum	Hydroxyapatite	Water	Tin
$K_{\alpha 1}$ (27.5)	.517	2.029	2.631	.418	8.974
$K_{\alpha 2}$ (27.2)	.264	2.085	2.706	.426	9.218
$K_{\beta 1}$ (31.0)	.140	1.467	1.882	.343	38.421
$K_{\beta 2}$ (31.7)	.031	1.387	1.774	.334	36.070
γ (35.4)	.049	1.051	1.329	.295	26.478

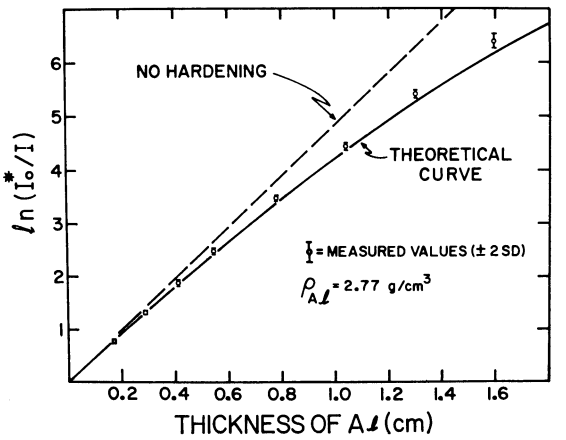

FIG. 1. The effect on the measurement of BMC due to hardening of the ^{125}I spectrum in mineral. For these measurements, the mineral was simulated by aluminum and the ^{125}I source was unfiltered. The $\ln (I_0^*/I)$ is a measure of bone mineral content.

on thickness of ± 0.0005 inch. The thickness of each of the three filters, as determined by an attenuation measurement using ^{241}Am for the photon source, was 26.03, 25.04, and 25.49 μm, respectively, with a standard deviation of 0.42 μm. A filter thickness of 25.4 μm has been assumed in all subsequent computations.

In all of the above investigations, measurements were made with the source and detector system stationary. Such point measurements avoid the edge effects which accompany scanning-type measurements made with a radiation beam of finite cross-section. The detection and analysis of the transmitted photons were accomplished using a NaI (Tl) scintillation spectrometer.* To observe the five components of the ^{125}I spectrum, the lower level of the pulse height analyzer was set at 7 kev and the window at 38 kev. These settings bracketed the photopeak while minimizing equipment noise and background. The measurements were made in a narrow-beam geometry with both the source and detector being collimated by 3-mm apertures. The source aperture was located 2.5 cm above the center of the source and the detector collimator extended 3.7 cm below the face of the NaI(Tl) crystal; the source-to-detector distance was 21 cm. With this geometry, energy shifts due to Compton scattering could be ignored in deriving Equation 1. The maximum angle through which a photon could be singly scattered and still enter the detector was 12°. A 30 kev photon would lose 40 ev of energy in such an interaction. The resulting change of the mass attenuation coefficients for water, aluminum, and hydroxyapatite would be $< 0.5\%$, which is negligible for this application.

Equation 1 was found to be a valid model for predicting the hardening effects due to both bone mineral and soft tissue (Figs. 1, 2). Figure 1 shows $\ln (I_0^*/I)$ as a function of thickness of aluminum, illustrating the nonlinearity in the relation between $\ln (I_0^*/I)$ and the mass of aluminum in the beam path. The points are measured values using an unfiltered ^{125}I source. The effect on $\ln (I_0^*/I)$ of increasing the soft-tissue cover is shown in Fig. 2, a plot of $\ln (I_0^*/I)$ as a function of the thickness of lucite covering the section of

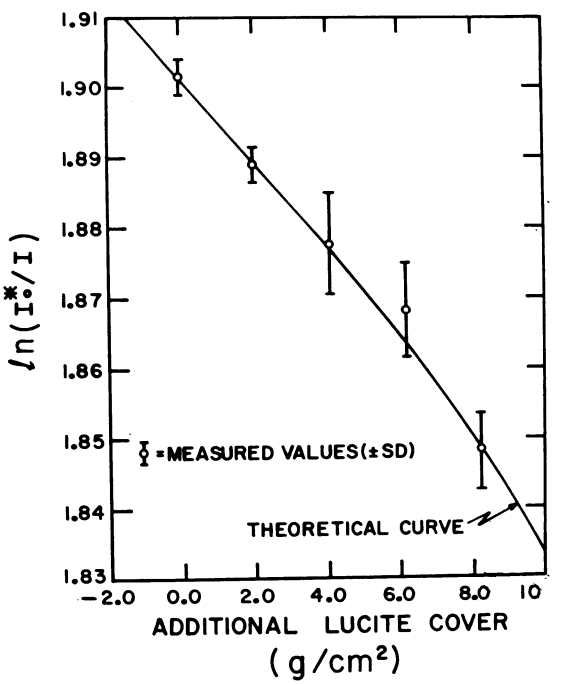

FIG. 2. The effect on the measurement of BMC due to hardening of the ^{125}I spectrum in soft tissue. Soft tissue variations were simulated by varying the amount of lucite over a section of bovine bone. The ^{125}I source was filtered with 50 μm of tin. If there were no hardening effect, the curve would be horizontal.

* Baird-Atomic Nuclear Spectrometer Model 530, Baird-Atomic, Inc., 125 Middlesex Turnpike, Bedford, Mass.

bovine bone. The source was filtered with 50 μm of tin.

Effect of Hardening on Point Measurements

Since it is common practice to relate $\ln (I_o^*/I)$ linearly to the bone mineral mass M_{BM}, g/cm^2, computations were performed to approximate the magnitude of the error introduced into point determinations of M_{BM} by the nonlinearity in the relationship between this quantity and $\ln (I_o^*/I)$. A linear estimator $<\ln (I_o^*/I)>$ of $\ln (I_o^*/I)$ was obtained by fitting a linear regression to Equation 1 over the range of M_{BM} from 0.45 g/cm^2 to 1.5 g/cm^2 of hydroxyapatite (HA). The total tissue thickness, t_{ST}, was chosen to be 7 cm and the amount of tin filtration was 51 μm, a thickness commonly used in BMC measurements.* This procedure would represent a calibration of a bone mineral measurement system under the above set of conditions. The resulting relation was

$$< \ln (I_o^*/I) > = 2.103 \times M_{BM}\,(g/cm^2) + 0.102 \quad (2)$$

with a standard error of estimate of 0.017. The ratio $R = < \ln (I^*_o/I) > / \ln (I_o/I)$ was used to quantify the above mentioned error. In this ratio, $< \ln (I_o^*/I) >$ was calculated using Equation 2 and $\ln (I_o^*/I)$ was calculated using Equation 1.

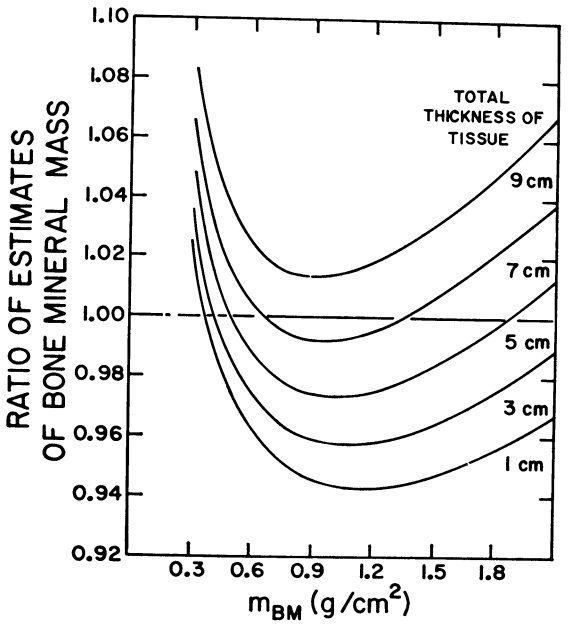

FIG. 3. Comparison of a particular linear estimator of BMC and Equation 1 (see text). Values on the ordinate are ratios of $<\ln (I_o^*/I)>/\ln (I_o^*/I)$. Abscissa values represent thicknesses of hydroxyapatite.

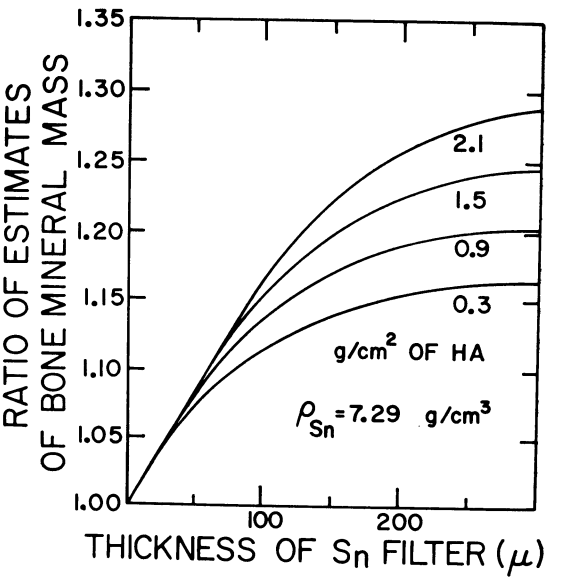

FIG. 4. The effect of variation of tin filtration on measurement of BMC. Values on ordinate are ratios of $\ln (I_o^*/I)$ calculated with tin filtration to the same quantity calculated without tin filtration.

Since Equation 2 has no explicit dependence on the value of t_{ST}, it is insensitive to changes of the amount of soft-tissue cover. However, $\ln (I_o^*/I)$ decreases as t_{ST} increases as is evident from Fig. 2. Consequently, changes in the thickness of soft-tissue cover can be misinterpreted as changes of BMC. To illustrate this effect, values of R were computed as a function of M_{BM} (g/cm^2 of HA) for several values of t_{ST}. The resulting curves are shown in Fig. 3. For a fixed value of M_{BM}, R increases by 0.5 to 1.0% per centimeter increase of soft-tissue cover, corresponding to a decrease of $\ln (I_o^*/I)$ also of 0.5 to 1.0%. For a system in which BMC is derived from $\ln (I_o^*/I)$ by a linear relation, the BMC would appear to decrease by 0.5 to 1.0%.

*AECL ^{125}I source Model C-236 supplied with 0.002 in (51 μm) tin filtration; Atomic Energy of Canada Limited, P. O. Box 6300, Postal Station J, Ottawa, Canada.

Norland Associates ^{125}I source Model 178A476A supplied with 0.0023 $\pm$ 0.0001 in (58.4 $\pm$ 2.5 μm filtration; Norland Associates, Fort Atkinson, Wis.

Also apparent from Fig. 3 is the hardening effect due to HA. For $M_{BM} \geq 1.2$ g/cm², and for the range of t_{ST} from 3 cm to 9 cm, R increases at the rate of 3.3% to 5.5% per g/cm² of HA. In particular, for the case $t_{ST} = 7$ cm, the calibration condition, the curve rises by 5% per g/cm² of HA. In the range of M_{BM} from 0.6 g/cm² to 1.5 g/cm² of HA, R is constant to within ± 1% for all values of t_{ST}. Over this range of M_{BM}, the linear estimator provides an accurate determination of BMC. In the region of $M_{BM} \leq 0.6$ g/cm² R increases rapidly since $< \ln (I_0^*/I) >$ approaches a finite value while $\ln (I_0^*/I)$ approaches zero as M_{BM} approaches zero.

Effect of Hardening on a Scan Measurement

The above comparison of a linear estimator of $\ln (I_0^*/I)$ and Equation 1 was extended to yield the magnitude of the error due to hardening effects to be expected in a scan measurement. In this case, values of $\ln (I_0^*/I)$ and $< \ln (I_0^*/I) >$ were calculated at 1-mm intervals across a hypothetical upper arm to obtain values of $\Sigma \ln (I_0^*/I)$ and $\Sigma < \ln (I_0^*/I) >$ for a cylindrical humerus. Measurements of 17 excised humeri indicated that an average outer diameter is 1.8 cm and an average inner diameter is 0.9 cm in the midshaft region. Calculations were made assuming values of $t_{ST} = 7$ cm and $t_{ST} = 8$ cm. With the assumption that compact bone is composed of 1/3 HA and 2/3 soft tissue, by volume, values of M_{BM} were computed which ranged from 0.82 g/cm² at 1 mm from the geometrical edge of the bone to 1.5 g/cm² at 5 mm from the center of the bone. A calculation of $\Sigma < \ln (I_0^*/I) >$ using Equation 2 and of $\Sigma \ln (I_0^*/I)$ using Equation 1 yielded the ratio $\Sigma < \ln (I_0^*/I) > / \Sigma \ln (I_0^*/I) = 1.001$ for $t_{ST} = 7$ cm, the calibration condition; for $t_{ST} = 8$ cm, the ratio was 1.012. This example indicated that errors due to hardening can be minimized by calibrating the system for the same range of bone mass values over which later measurements are to be made. Also, since variation in total tissue thickness can introduce an error as large as 1% per centimeter deviation from the calibration tissue thickness, standardization of this thickness will maximize the precision and accuracy of the measurement.

Effects of Tin Filtration on the BMC Measurement

The calculations of the previous section were performed under the assumption that there were 51 μm of tin filtration in the beam. Tin is the element of choice because it has a K-absorption edge at 29.2 kev. Hence tin filtering makes the ^{125}I beam more nearly monoenergetic by selective attenuation of the $K_{\beta 1}$ (31.0 kev), and $K_{\beta 2}$ (31.7 kev), and the γ (35.0 kev) radiation. However, variations of the thickness of tin filtration in the beam can lead to considerable errors in the BMC measurements. Computations using Equation 1 indicated that the magnitude of $\ln (I_0^*/I)$ increases with increasing tin filtration at the rate of 0.12% to 0.16% per μm of tin over the range of 0 to 76 μm of tin filtration and with absorber thicknesses of 0.3 to 2.1 g/cm² of HA. Measurements on the aluminum stepwedge over the same range of tin filtration and a similar range of absorber thickness showed an increase in $\ln (I_0^*/I)$ of 0.12%, to 0.20% per μm of tin. This effect is displayed in Fig. 4, which shows the ratio of $\ln (I_0^*/I)$ calculated with tin filtration to the same quantity calculated without filtration as a function of the thickness of tin filtration and for several thicknesses of HA.

Discussion

In order to hold the variation of measured BMC due to changes of tin filtration to ≤ 1% the thickness of filters used with different sources should be controlled to within ± 2.5 μm. Since ^{125}I has a 60-day half life, three different sources would be used per year in the typical clinical situation. One method of maintaining constant filter thickness is to use the same filter on each ^{125}I source used for BMC measurements. Since such a filter would be subject to considerable handling, it should be protected, possibly by laminating it be-

tween thin sheets of lucite. Uniformity of thickness among several filters could be achieved by making transmission measurements on the filters and selecting those for which the transmitted count rates are within the required limits.

The problems of tin filter variations can be eliminated entirely by not using any filter. Although this tactic will lead to a more heterochromatic beam, the hardening effects can be accounted for by calibrating the system over the range of BMC values to be encountered in practice and by maintaining the same total tissue thickness for all measurements. For example, a linear regression, similar to that described previously, was determined for $\ln(I_o^*/I)$ as a function of M_{BM}, but in this case no tin filtration was assumed. Computation of $\Sigma < \ln(I_o^*/I) >$ and $\Sigma \ln(I_o^*/I)$ for the hypothetical upper arm scan produced the ratios $\Sigma < \ln(I_o^*/I) > / \Sigma \ln(I_o^*/I) = 0.997$ with 7 cm tissue thickness, the calibration thickness, assumed, and 1.010 with 8 cm tissue thickness assumed.

Eliminating the filter will also provide a considerable increase of the photon fluence. For example, our use of 51 μm of tin filtration reduces the count-rate by $\sim 50\%$. Removing the tin filter (and also broadening the spectrum accepted by the pulse height analyzer) will permit longer use of present sources or the purchase of lower activity, less expensive sources for future use. However, such changes would have to be accompanied by a recalibration of the BMC measurement system.

Conclusion

Hardening effects were found to be sufficiently small that a linear relation between BMC and $\ln(I_o^*/I)$ could predict the BMC to within $\pm 1\%$. However, in order to maintain this accuracy the measuring apparatus must be calibrated over the same range of BMC values that will be encountered in practice. Significant errors in accuracy, $\sim 5\%$ per g/cm^2 of HA, can be introduced into measurements of BMC outside the range of calibration. Although the errors due to variations in soft-tissue cover were small, $\leqslant 1\%$ per g/cm^2 of soft tissue, control of the total tissue thickness will maintain the accuracy and increase the precision of the measurement. Finally, errors affecting accuracy and precision can be minimized by control of the amount of tin filtration used in the beam. These errors can be eliminated by elimination of the tin filter or at least minimized by quality control of the thickness of the filters used on the ^{125}I sources.

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